New Solutions to the Hierarchy Problem

What to Expect at the TeV Scale

Lecture II

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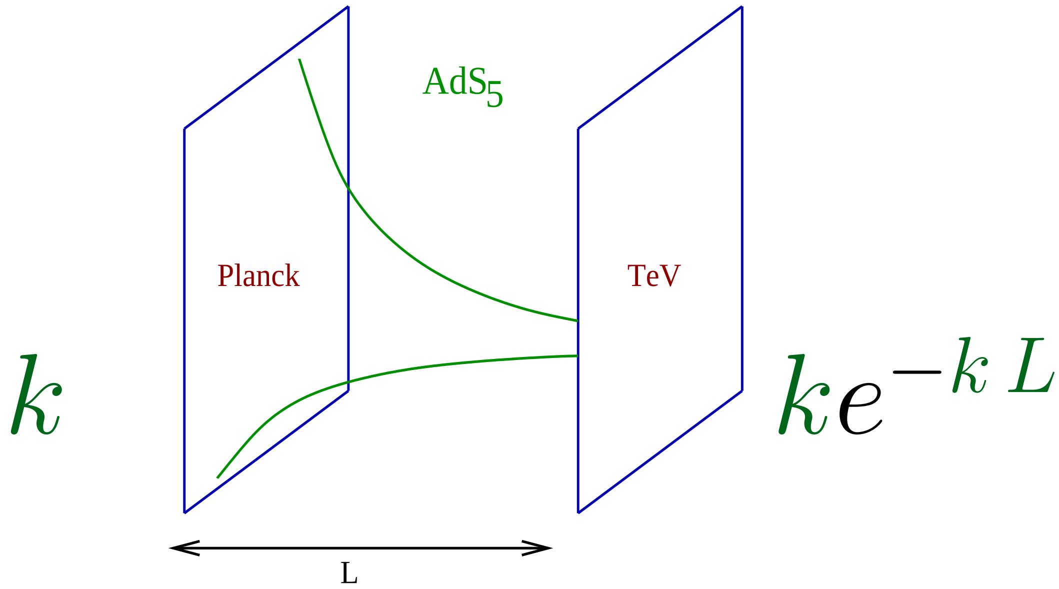
Gravity in Extra Dimensions

Can we really expect the extra dimensions to be flat ?

- The Cosmological Constant (CC) in 4D is small but non-zero. What about in XD ?
- To stabilize XD with small CC in 4D slices (our universe) may require a large CC in the bulk. This fine-tuning of the CC is just the CC problem.
- But sizeable CC in bulk $\Rightarrow$ bulk is not flat. How curved is the bulk ?
- Model-dependent question: How is the XD stabilized ?
- We opened Pandora's box: Curved extra dimensions.
- Maximally symmetric open metric: Anti-De Sitter. Also motivated by AdS/CFT string-gauge duality (Maldacena '98).

Warped Extra Dimensions

- One compact extra dimension. Non-trivial metric induces small energy scale from Planck scale! (L. Randall, R. Sundrum).



- Geometry of extra dimension generates hierarchy exponentially!

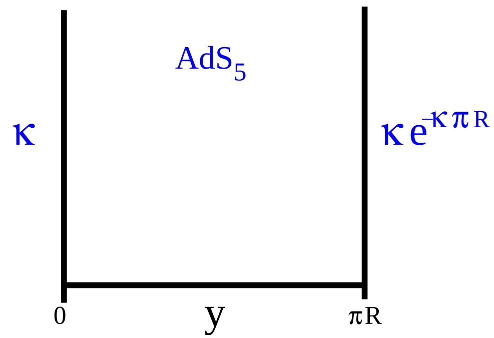
$$\boxed{\Lambda_{\mathrm{TeV}} \sim M_{\mathrm{Planck}}\, e^{-k\,L}}$$ with k the curvature

Warped Extra Dimensions

- Warped 5D metric in RS

$$ds^2 = e^{-2\kappa|y|}\,\eta^{\mu\nu}dx_\mu dx_\nu + dy^2$$

- Compactified on S_1/Z_2 with $L = \pi R$



and $k \lesssim M_P$, AdS_5 curvature.

- For $kR \simeq (11 - 12)$

$$\longrightarrow \kappa\, e^{-\kappa\pi R} \simeq O(\mathrm{TeV}).$$

Warped Extra Dimensions

Solving the Hierarchy Problem:

- If Higgs localized at $y = \pi R$

$$S_H = \int d^4x \int_0^{\pi R} dy\, \sqrt{-g}\, \delta(y - \pi R) \left[g_{\mu\nu} \partial^\mu H^\dagger \partial^\nu H - \lambda \left(|H|^2 - v_0^2 \right)^2 \right]$$

- Warp factors e^{ky} appear in $g_{\mu\nu}$ and $\sqrt{-g}$.

$$S_H = \int d^4x \, \left[e^{-2k\pi R} \eta_{\mu\nu} \partial^\mu H^\dagger \partial^\nu H - e^{-4k\pi R} \lambda \left(|H|^2 - v_0^2 \right)^2 \right]$$

- Canonically normalize $H \Rightarrow$

$$e^{-k\pi R} H \to H$$

Warped Extra Dimensions

What we get now is

$$S_H = \int d^4x \left[\eta_{\mu\nu}\partial^\mu H^\dagger \partial^\nu H - \lambda\left(|H|^2 - e^{-2k\pi R} v_0^2\right)^2\right]$$

$\Rightarrow$ If $v_0 \simeq M_P$, then “new”scale in exponentially suppressed

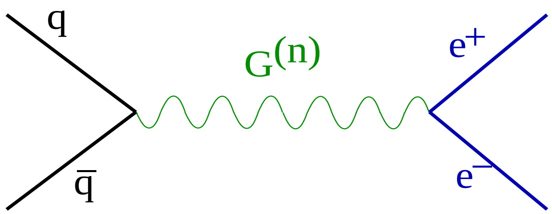
$$v = e^{-k\pi R}\, v_0$$

Choosing $kR \simeq O(10)$ gets us $v \simeq$ weak scale.

Warped Extra Dimensions

If only gravity propagates in the bulk (SM fields on TeV brane):
$\Longrightarrow$ Kaluza-Klein graviton tower

- Zero-mode graviton $G^{(0)}$ localized toward the Planck brane. This is why gravity is weak! $G^{(0)}$ couples to SM fields as $1/M_P^2$
- First few KK graviton excitations localized toward TeV brane $\to$ They couple strongly (as $(1/\mathrm{TeV})^2$ to fields there.
  E.g.: Drell-Yan at hadron colliders



Bulk Life in WED

- In original proposal, *only gravity* propagates in 5D bulk.
- RS is a solution of the hierarchy problem. But origin of EWSB? And flavor ? ...
- Allowing gauge fields and matter to propagate in the bulk opens many possibilities: models of EWSB, GUTs, flavor, ...
- The 5D mass of a bulk fermion $=>$ *localization* of zero-mode.
- If Higgs remains on TeV brane:
  Fermions localized toward TeV brane are more massive
  Fermions localized toward the Planck brane are lighter

$\Longrightarrow$ $\boxed{\text{Fermion Geography}}$

Gauge Fields in the 5D bulk

- Gauge Fields in the 5D bulk: KK decomposition in 4D (for $A_y = 0$ gauge):

$$A_\mu(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} A_\mu^{(n)}(x)\chi^{(n)}(y) \ ,$$

  Zero-mode $A_\mu(x)^{(0)}$ + KK tower of massive gauge bosons for $n > 0$,
  with masses

$$m_n \simeq (n - O(1)) \times \pi\kappa e^{-\kappa\pi R}$$

  I.e. for appropriate choice of κR 1st KK excitations are $O(\mathrm{TeV})$.

- 1st KK excitations have $\chi^{(n)}(y)$ localized toward TeV fixed point.

- The Gauge Symmetry usually either is or embeds the SM:
  e.g. $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X$ (restores custodial symm.)

WED and Flavor

- Fermion Fields in the bulk: 5D fermion field KK decomposition

$$\Psi_{L,R}(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0} \psi_n^{L,R}(x) e^{2\kappa|y|} f_n^{L,R}(y)$$

- 5D fermion bulk mass term $\longrightarrow$ localization of fermion fields:

$$S_f = \int d^4x\, dy\, \sqrt{-g}\, \{\cdots - \; c\, \kappa \bar{\Psi}(x,y)\Psi(x,y)\} \; ,$$

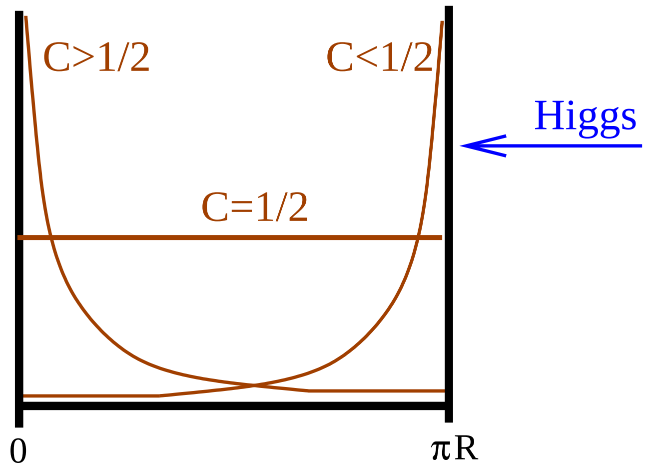
with $c \simeq O(1)$.

- $=>$ Fermion zero-modes can be localized by choosing c :

$$f_0^{R,L}(y) = \sqrt{\frac{\kappa\pi R\,(1 \pm 2c)}{e^{\kappa\pi R(1 \pm 2c)} - 1}}\; e^{\pm c\,\kappa\, y}$$

Flavor Models in WED

- $O(1)$ flavor breaking in bulk can generate fermion mass hierarchy:

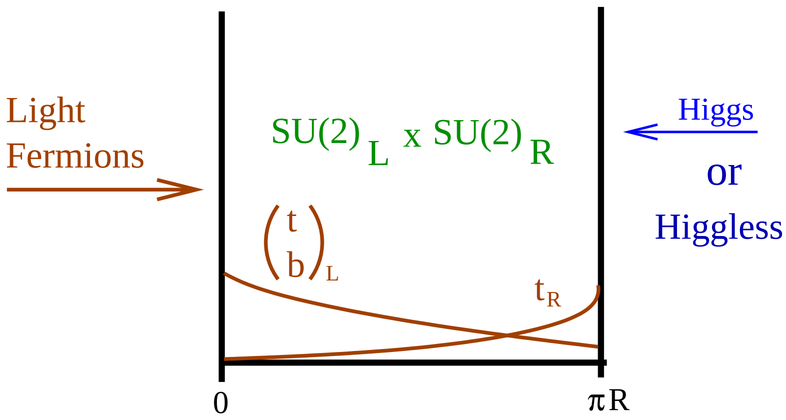


Fermions localized toward the TeV brane can have larger Yukawas, Those localized toward the Planck brane have highly suppressed ones.

- But fermions at $\simeq \pi R =>$ strong couplings to 1st KK gauge bosons!
E.g: 3rd generation quarks might have large couplings $\rightarrow$ flavor violation.

Electroweak Symmetry Breaking and WED

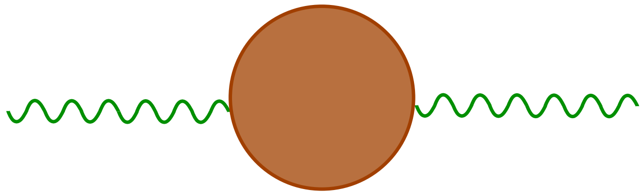
Several possibilities for model building:



- Higgs or Higgless (BC breaking)
- Light fermions on Planck brane or off (reduces S)
- Third generation remains a challenge

Electroweak Symmetry Breaking and WED

- EW Precision constraints: potentially large corrections to gauge boson propagators



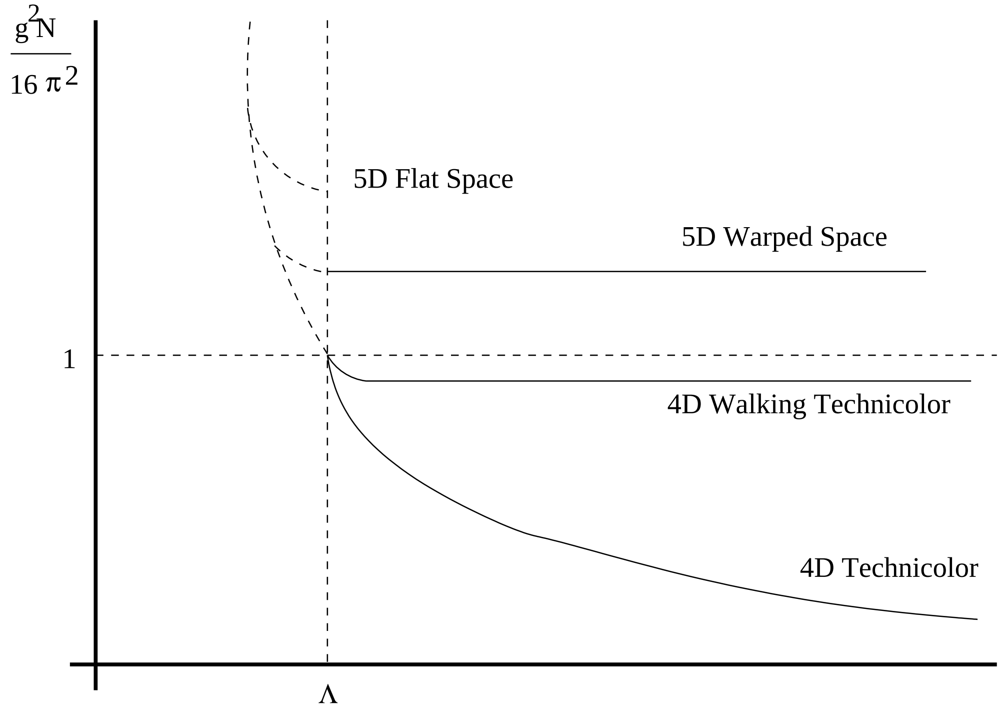
$$S \sim \frac{N}{\pi} = 16\pi \frac{v^2}{m_{KK}^2}$$

- Important constraints for building models:
  Canceling the effect of S vs. small N.

Warped Extra Dimensions - Signals

- Narrow states $\rightarrow$ KK modes (Large N).
  Spin 2 (graviton), possibly all other SM fields.
- No spaced resonances, but broad enhancements in cross sections (Small N).
- Flavor violation at tree level $\rightarrow$ Potentially rich array of deviations:
  Effects of KK gluons in CP asymmetries in $B \rightarrow \phi K_s$, $B \rightarrow \pi K_s$, B_s mixing, $\ldots$;
  Z mixing with KK excitations in $b \rightarrow s\ell^+\ell^-$
- Conclusion: Back to strong dynamics at the TeV scale. Can the extra dimensional picture help ?

Warped Extra Dimensions vs. Strong Dynamics



Little Higgs Mechanism

Solve the Hierarchy problem: keep the Higgs light, keeping the effects of the quantum corrections m_h^2 to a minimum.

- One way: If Higgs is (part of) a Nambu-Goldstone boson (NGB) then it cannot have a mass. (It has only derivative interactions).
- In the SM: $SU(2)_L \times U(1)_Y$ spontaneously broken to $U(1)_{EM}$

  $$\#\text{NGBs} = \#\text{broken generators} = 3$$

  But since the gauge symmetry is broken $\Rightarrow$ the 3 NGBs are eaten.
- $\Rightarrow$ to have NGBs left over, need that

$$\boxed{\textit{Global}\ \text{Symmetry} > \text{Gauge Symmetry}}$$

Little Higgs Mechanism

- The spontaneous breaking of a *Global* symmetry would generate NGBs.
  We need that one of them be a $SU(2)_L$ doublet (H).
- Problem: We do not want a *massless* Higgs. NGBs are exactly massless.
- Need some form of explicit breaking so that H gets a mass (pseudo-NGB).

Little Higgs Theories

- Example: π is a pNGB of $SU(2)_L \times SU(2)_R / SU(2)_V$.
  - # of NGBs = 3 + 3 -3 = 3
  - The NGB $\vec{\pi} = (\pi^+, \pi^-, \pi^0)$ acquires a mass due to explicit breaking: quark masses $\neq 0 \Rightarrow$

$$m_\pi^2 = B_0 m_q$$

- Another Example:
  - $SU(2)_V \;\Rightarrow m_{\pi^\pm} = m_{\pi^0}$
  - But QED (i.e. $U(1)_{EM}$) explicitly breaks $SU(2)_V$ giving a mass difference (Dashen's formula).

Simplest Little Higgs Theory

Ingredients:

- Global symmetry: $SU(3)/SU(2)$?
  If we gauge part of this with SM gauge group still get quadratic divergences ! E.g. $SU(2)_L$ explicitly breaks global $SU(3)$.
- Use $SU(3)$ gauge symmetry ?
  This respects $SU(3)$ global. But when we break the global symmetry, we break the gauge symmetry $\Rightarrow$ *all* NGBs are eaten!
- Solution: Enlarge global symmetry to $(SU(3))^2$.

$$\mathcal{L} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2$$

with Φ_1 and Φ_2 NGBs from $(SU(3)/SU(2))^2$, and there is no potential at tree level.

Simplest Little Higgs Theory

The NGBs can be written as

$$\Phi_1 = e^{i\pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix} \qquad , \Phi_2 = e^{-i\pi/f} \begin{pmatrix} 0 \\ 0 \\ f \end{pmatrix}$$

with

$$\pi = \pi^a t^a = \eta/\sqrt{2} + \begin{pmatrix} 0 & 0 & h_1 \\ 0 & 0 & h_2 \\ h_1^* & h_2^* & 0 \end{pmatrix}$$

and

$$\begin{pmatrix} h1 \\ h2 \end{pmatrix} = h$$

is the $SU(2)_L$ doublet we call the Higgs.

Simplest Little Higgs Theory

Gauge Symmetry: $SU(3)$ (contains the SM $SU(2)_L \times U(1)_Y$)

- It respects

$$\mathcal{L} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2$$

  Both global $SU(3)$'s respected separately.

- Gauge symmetry does lead to quadratically divergent terms

$$\frac{g^2}{16\pi^2}\Lambda^2 \left(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2\right) = \frac{g^2}{16\pi^2}\Lambda^2 \left(f^2 + f^2\right)$$

  but it does not generate a $h^\dagger h$ term !

Simplest Little Higgs Theory

- But at one loop we can generate operators like $|\Phi_1^\dagger \Phi_2|^2$, respecting the $SU(3)$ gauge symmetry, but generating a Higgs mass term.
- However, these are only log divergent:

$$\frac{g^4}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) |\Phi_1^\dagger \Phi_2|^2$$

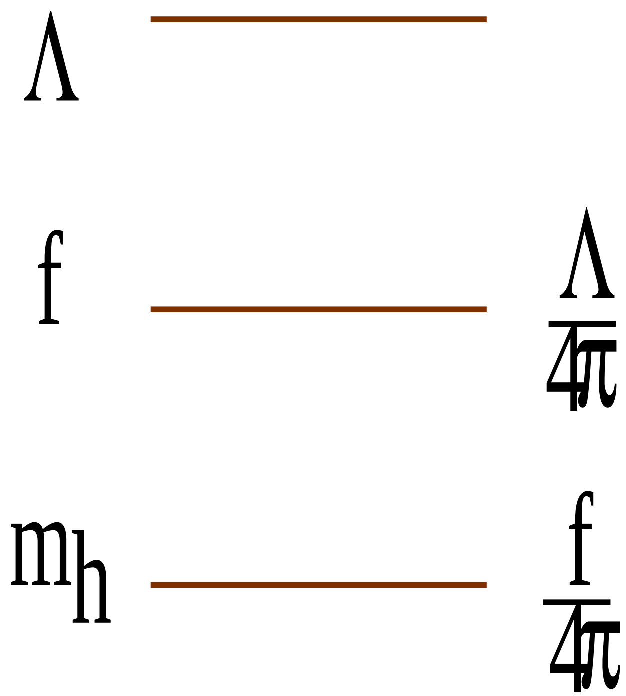
- This leads to a contribution to the Higgs mass

$$\delta m_h^2 \simeq \frac{g^4}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) f^2$$

that is $m_h \simeq f/4\pi$.

Simplest Little Higgs Theory

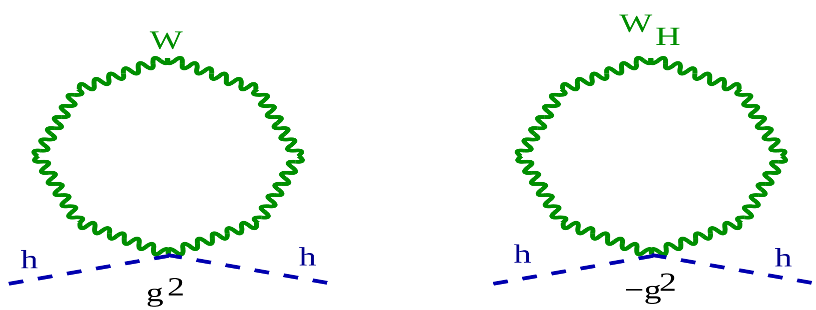
Energy Scales: $m_h \simeq f/4\pi$; $\Lambda \simeq 4\pi f$ (NDA)



Simplest Little Higgs Theory

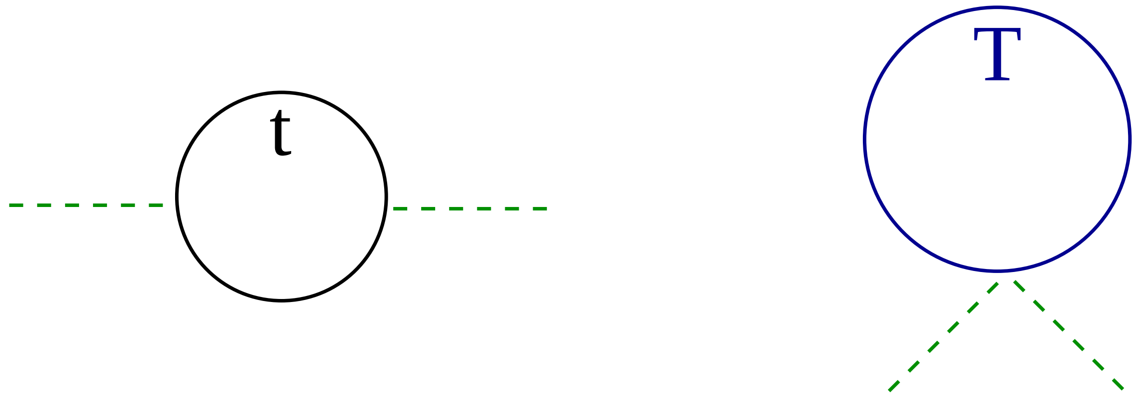
But where is the miracle of the cancellations ? Or What states cancel the top loop, the gauge boson loop and the Higgs loop contributions to δm_h^2 ?

- Since we have $SU(3)$ (instead of $SU(2)_L \times U(1)_Y$) we have additional gauge bosons that get masses of order f. They cancel the quadratic divergences from SM gauge loops.



Simplest Little Higgs Theory

- Also need to promote all fermion doublets to triplets of $SU(3)$.
  $\Rightarrow$ Additional fermions cancel quadratic divergences from SM fermion loops (particularly the top).



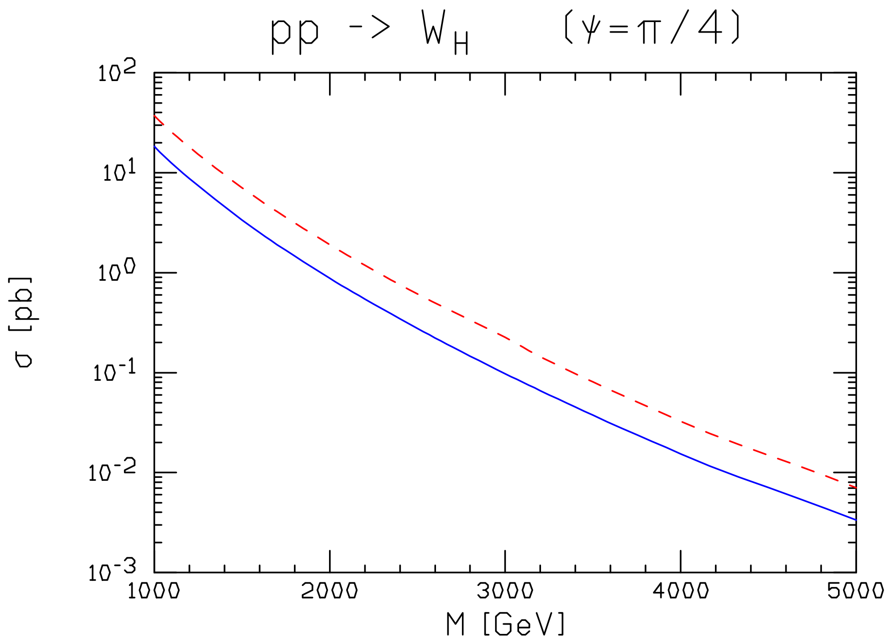
Little Higgs Theories

Many other models in the market:

- Littlest Higgs: $SU(5)/SO(5)$ with $SU(2) \times SU(2)$
- T Parity models: large amounts of new states, but nice agreement with EW precision constraints.
- Many more $\cdots$
- All of them result in new fermions (at least “heavy tops”) and new gauge bosons.
- If $f \simeq O(1)$ TeV, cutoff Λ is in the tens of TeV.
- Models make distinct predictions for production/decays of new particles.

Littlest Higgs Phenomenology

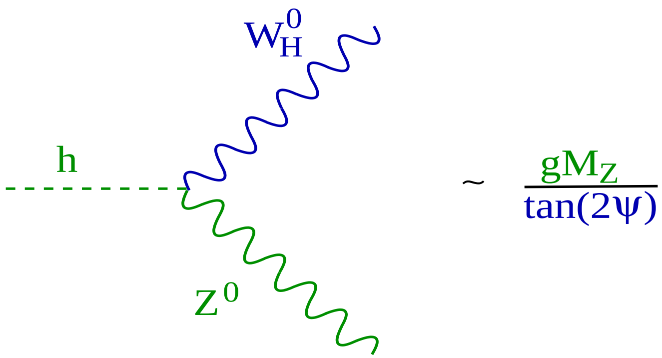
- LHC will have lot of new gauge bosons at 10^5/year !



- But many extensions of the SM predict extra gauge bosons ...
  How do we know is the Little Higgs ?

Littlest Higgs

- Little Higgs prediction: Symmetries dictate how the new gauge bosons couple to the Higgs (loop cancellations). $\Rightarrow$ need to measure this coupling.
- Strategy: study the decays of W_H



- Measuring the W_H production $\Rightarrow$ mixing angle ψ. Gives prediction for $(W_H \to Z\, h)$, test model.