Collider Physics

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PLAN

- I. Basic facts
- II. Probing the Standard Model
- III. Beyond the SM searches

First part: Basic Facts

- I. Colliders as tools
- II. Collider parameters
- III. $\mathbf{e^+e^-}$ colliders
- IV. Hadron colliders
- V. Detectors
- VI. Kinematics at hadron colliders
- VII. Evaluation of scattering amplitudes

I. Colliders as tools

- Relativity and quantum mechanics lead to

$$\Delta p \, \Delta t > \frac{\hbar}{c}$$

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- We can only observe asymptotic states

- This is like studying classical mechanics and how a car is made looking at

II. Collider parameters

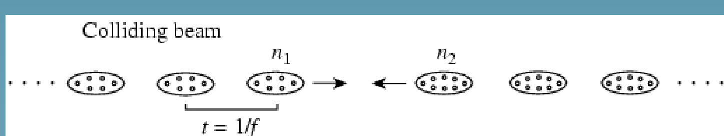
❄ Center-of-mass energy $1+2 \to X$

$$s \equiv E_{CM}^2 \equiv (p_1+p_2)^2 = \begin{cases} (E_1+E_2)^2 & \text{in the c.m. frame } \vec{p}_1+\vec{p}_2=0, \\ m_1^2+m_2^2+2(E_1E_2-\vec{p}_1\cdot\vec{p}_2). \end{cases}$$

❄ Instantaneous luminosity $\mathcal{L}$: the number of events/second is proportional to the cross section

$$N_{\text{events}} = \mathcal{L}\,\sigma(s)$$

- beams are a collection of bunches with n_i particles and crossing frequency f



$$\mathcal{L} \propto \frac{n_1 n_2 f}{a}$$

where a is the transverse area of the bunch.

❄ Useful collider's luminosity change of units

$$10^{33}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1} = 1\ \mathrm{nb}^{-1}\ \mathrm{s}^{-1} \approx 10\ \mathrm{fb}^{-1}/\mathrm{year}.$$

❄ e^+e^- colliders

Colliders	$\sqrt{s}$ (GeV) (GeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (kHz)	polar.	L (km)
LEP I	M_Z	2.4×10^{31}	$\sim 0.1\%$	45	55%	26.7
SLC	~ 100	2.5×10^{30}	0.12%	0.12	80%	2.9
LEP II	~ 210	10^{32}	$\sim 0.1\%$	45		26.7
BEPC	up to 4.4	$\sim 10^{31}$	$\sim 6. \times 10^{-4}$	1200		0.24
	(TeV)			(MHz)		
ILC	0.5–1	2.5×10^{34}	0.1%	3	$80, 60\%$	14-33
CLIC	3–5	$\sim 10^{35}$	0.35%	1500	$80, 60\%$	33-53

❄ A limiting factor is the energy loss $\Delta E \propto \frac{1}{R}\left(\frac{E}{m}\right)^4$,

❄ Hadronic colliders

Colliders	$\sqrt{s}$ (TeV)	$\mathcal{L}$ (cm^{-2}s^{-1})	$\delta E/E$	f (MHz)	#/bunch (10^{10})	L (km)
Tevatron	1.96	2.1×10^{32}	9×10^{-5}	2.5	p: 27, $\bar{p}$: 7.5	6.28
HERA	314	1.4×10^{31}	$0.1, 0.02\%$	10	e: 3, p: 7	6.34
LHC	14	10^{34}	0.01%	40	10.5	26.66
SSC	40	10^{33}	5.5×10^{-5}	60	0.8	87
VLHC	40–170	2×10^{34}	4.4×10^{-4}	53	2.6	233

❄ $\sigma \mathbf{propto} \mathbf{E}^2_{\mathbf{CM}}$ LRA $cal L2$ grows as $\simeq \mathbf{E}^2_{\mathbf{CM}}$

❄ LHC: time between collisions 25 ns.

❄ At luminosity 10^{34} cm^{-2} s^{-1} there will be 10^9 interactions/s $\Longrightarrow \simeq 25$ collisions per bunch crossing (pileup)

III. $\mathbf{e^+e^-}$ colliders

❄ Main advantages

- e^+e^- interactions are well understood in the SM.
- the e^+e^- systems has zero charge, lepton number, etc $\Longrightarrow$ good to produce new states.
- beam properties are well understood $\Longrightarrow$ scattering kinematics is well constrained.
- If CM coincides with the lab frame $\Longrightarrow$ the total $\sqrt{s}$ can be used.
- It is possible to polarize the initial beams.

❄ Main disadvantages

- Large synchrotron radiation $\Longrightarrow$ linear machines
- e^+e^- couples mainly to spin-1 states in the s-channel.
- The energy losses due to bremstrahlung
- At high energies there can be large energy losses due to beam-beam interaction (beamstrahlung).
- This leads to $R(s) = \int d\tau \frac{d\mathcal{L}}{d\tau}\, \sigma(\hat{s})$ with $\tau = \sqrt{\hat{s}}/\sqrt{s}$

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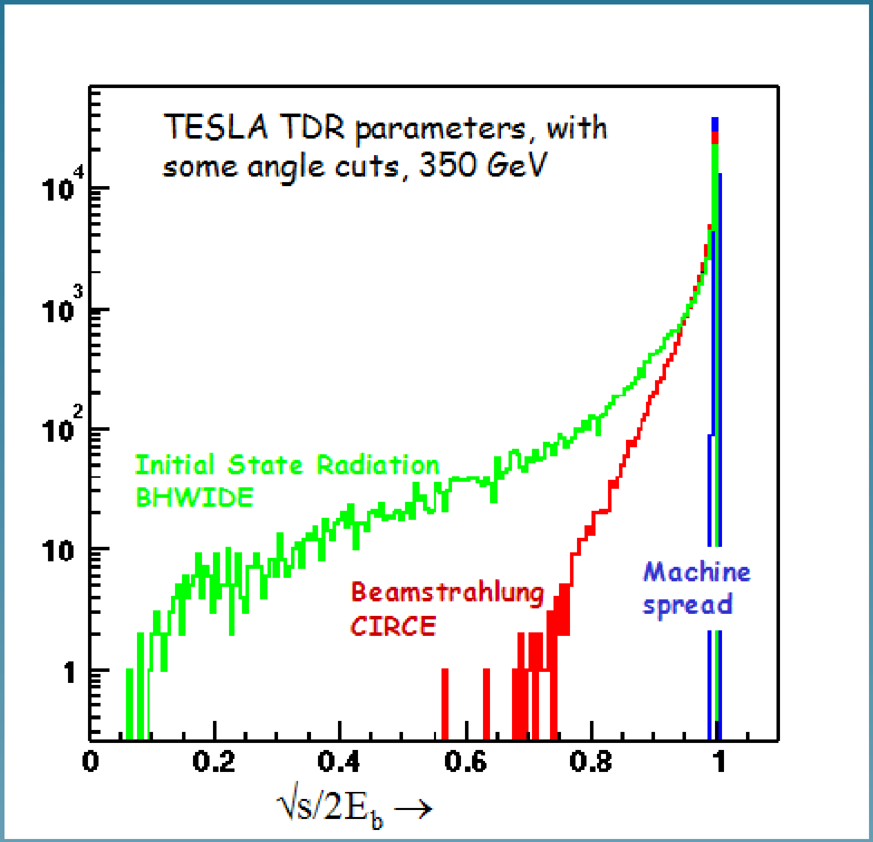

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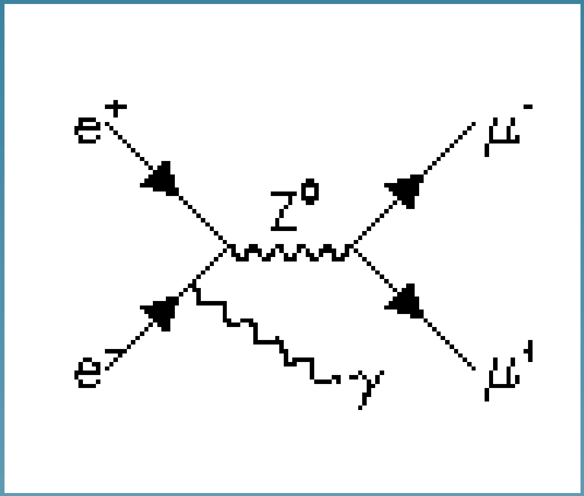

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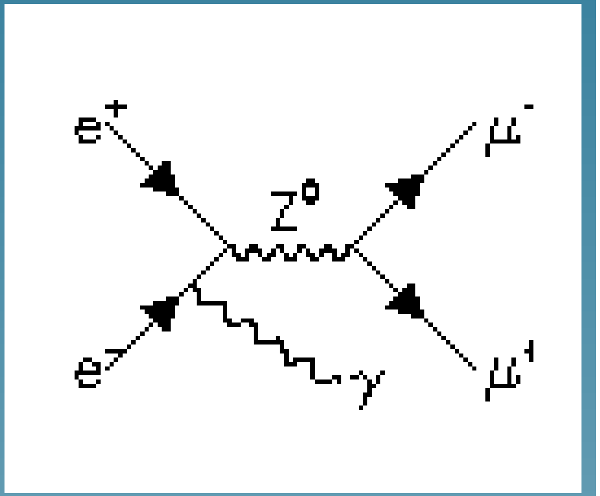
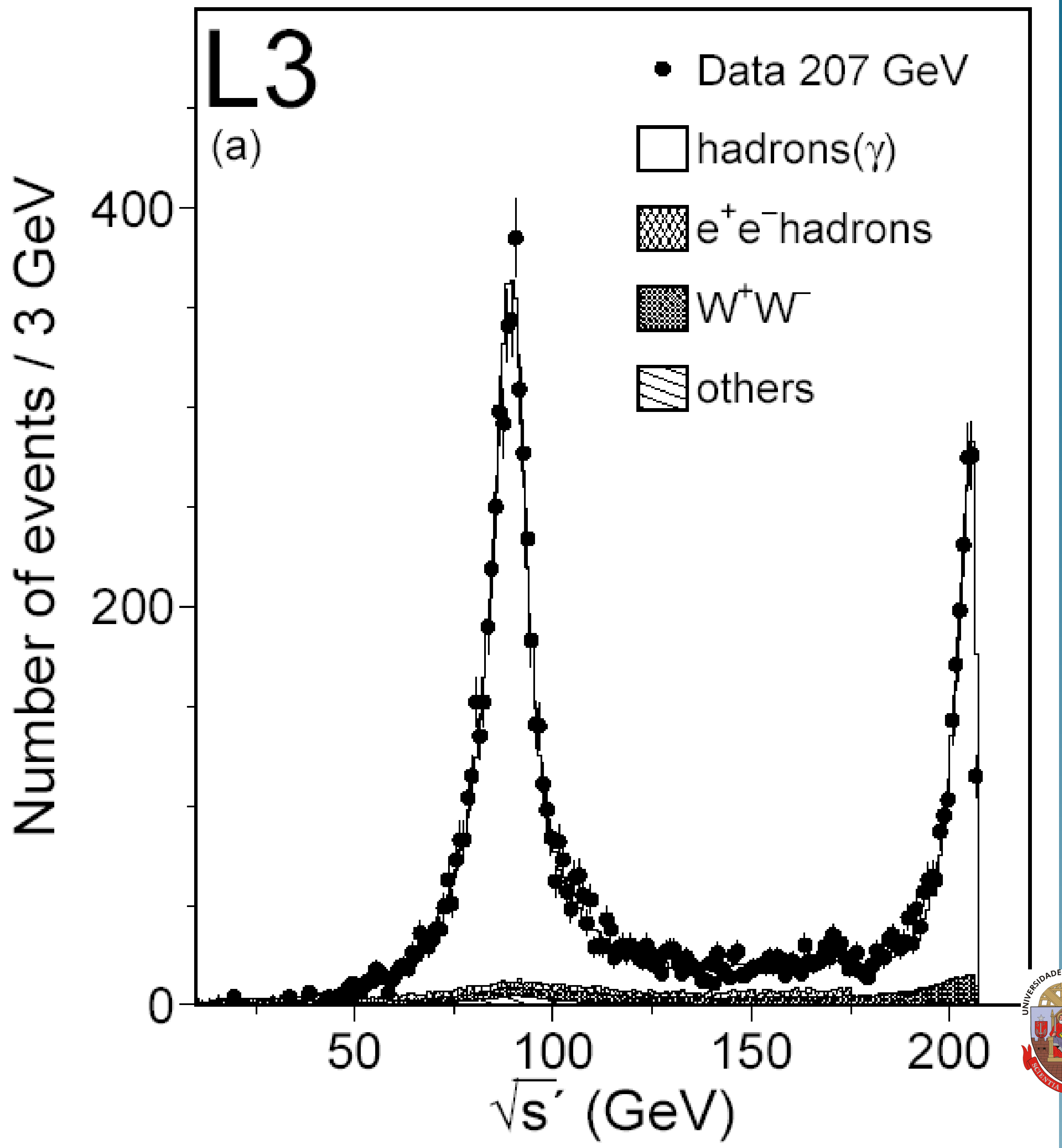
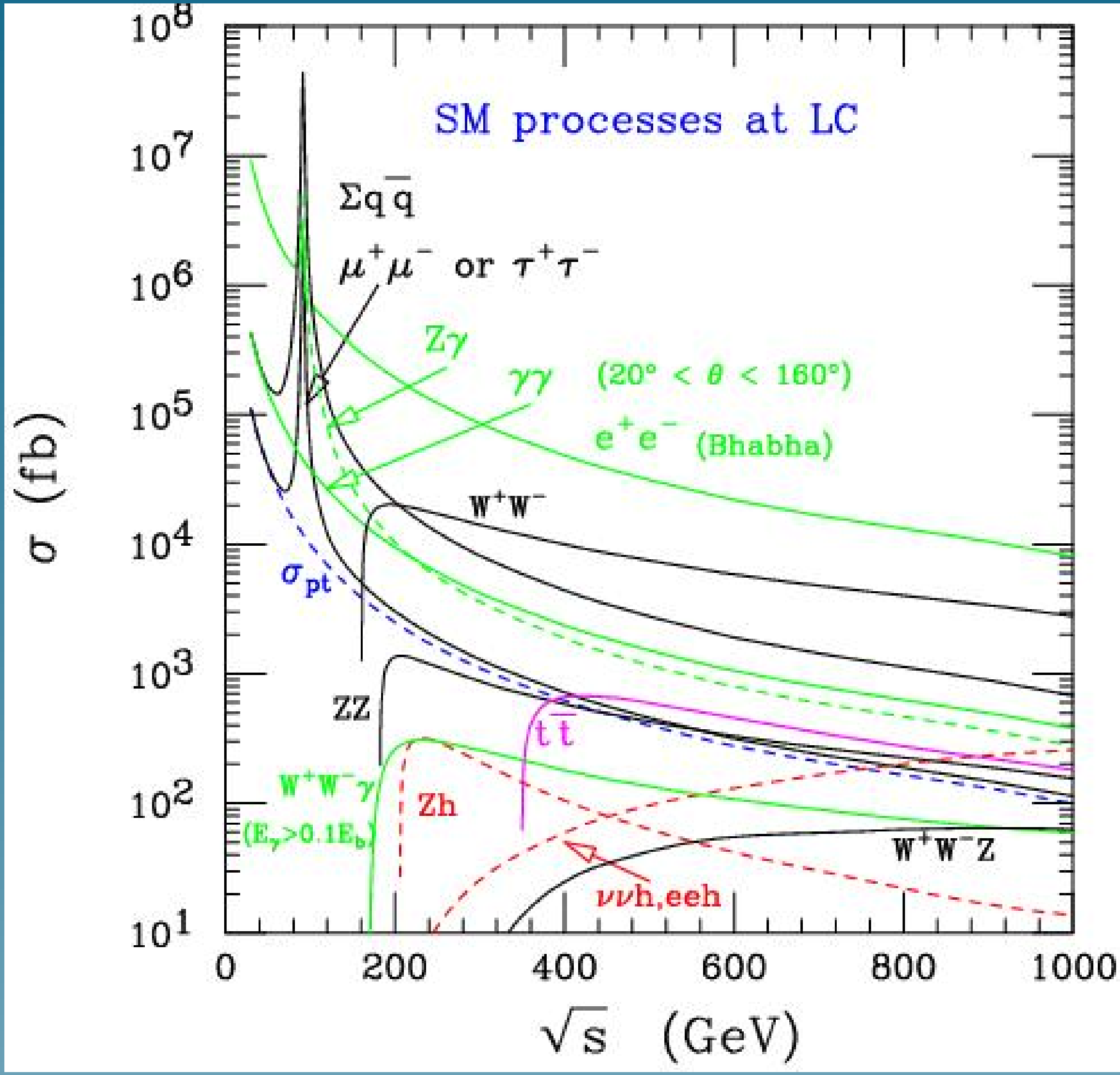




- Main processes



IV. Hadron colliders

❀ Protons are composite of quarks and gluons and they are much heavier than electrons.

❀ The main differences are

- Higher CM energies since $m_P >> m_e$
- Higher luminosities can be achieved
- Protons interact strongly $\Longrightarrow$ large cross sections ($\sigma_{total} \simeq 100$ mb).

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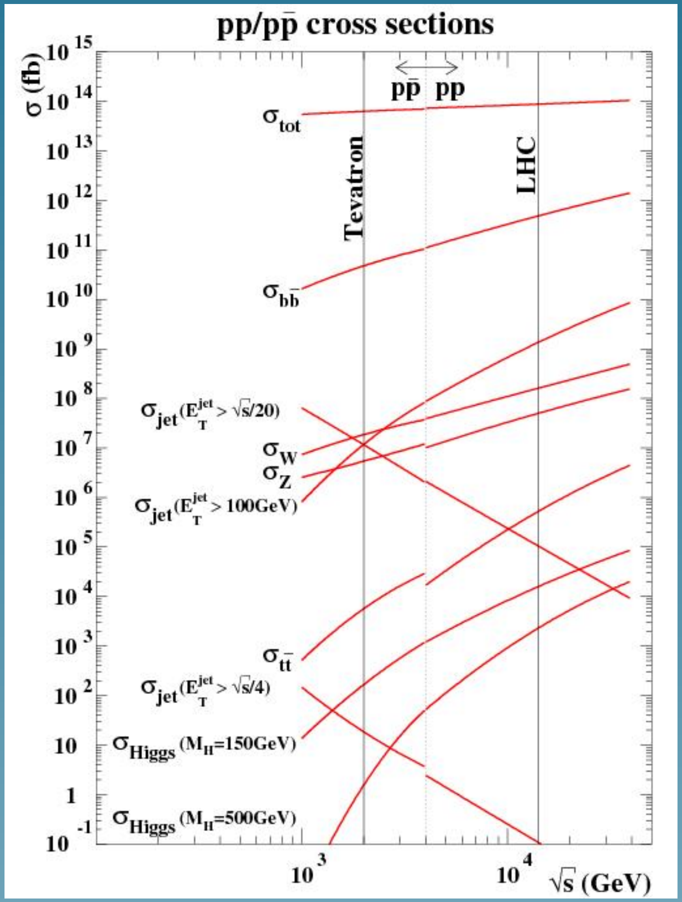
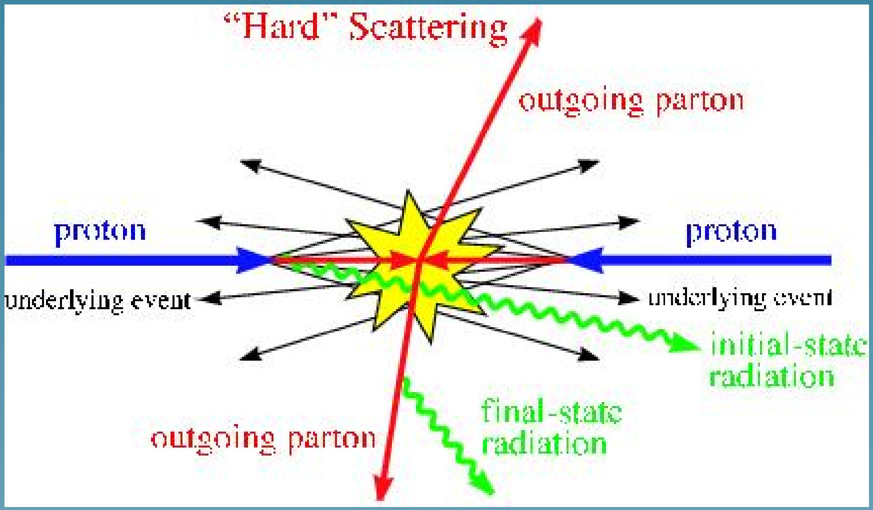
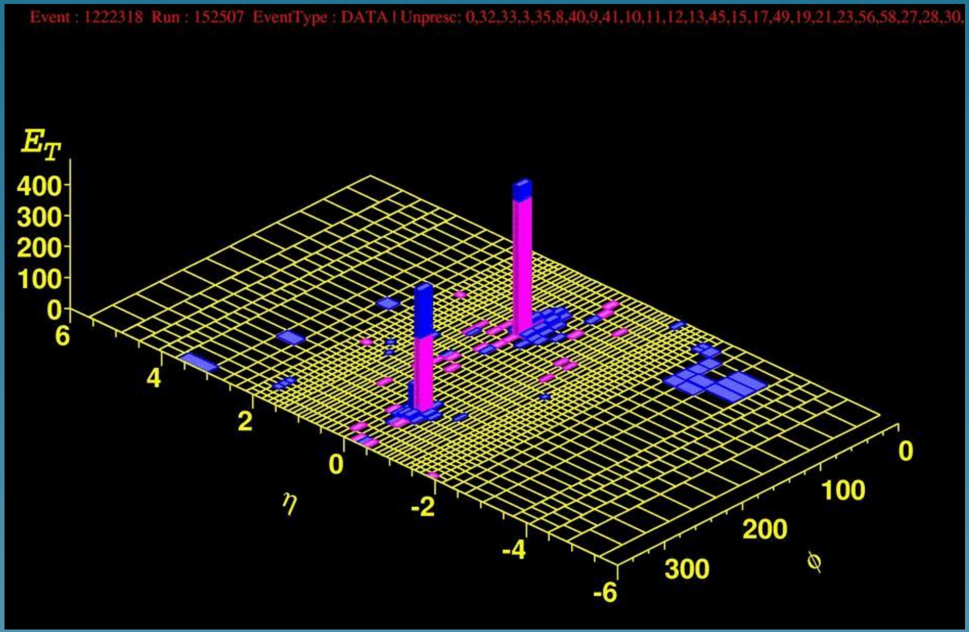


- The collisions are messier



- Dijet event at CDF



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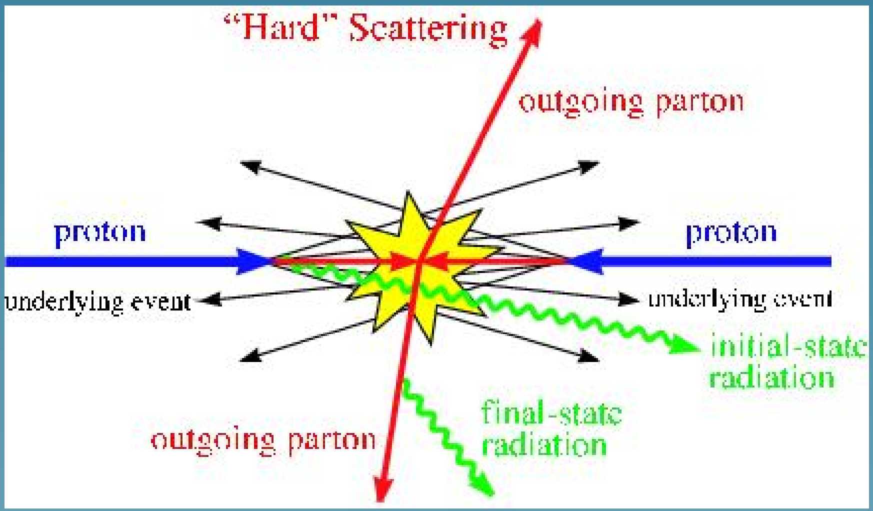

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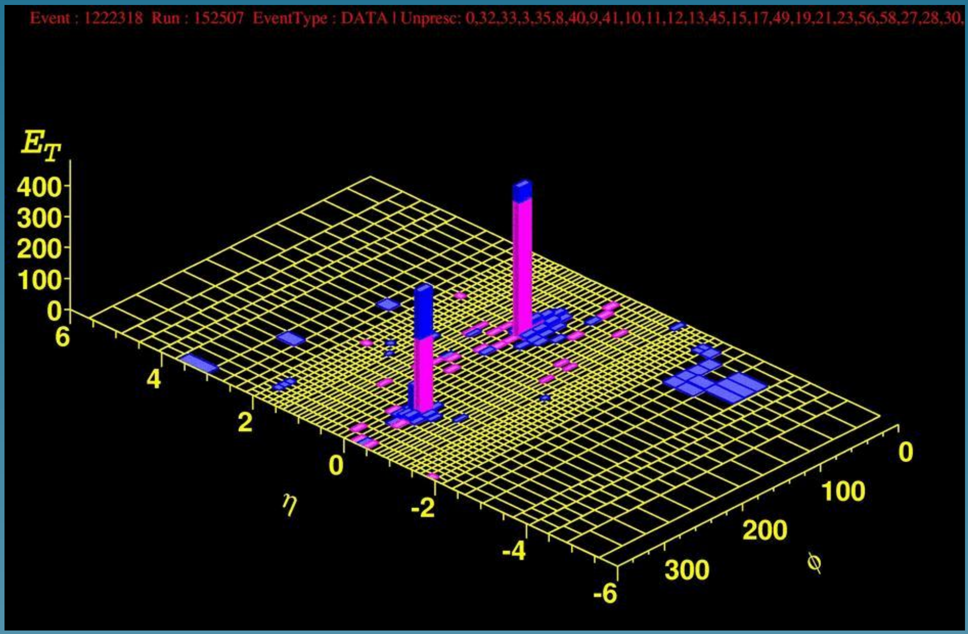

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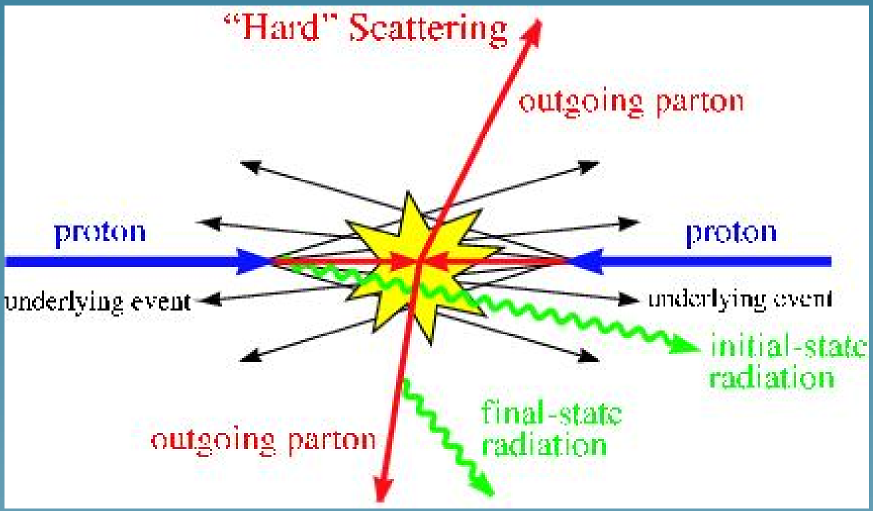

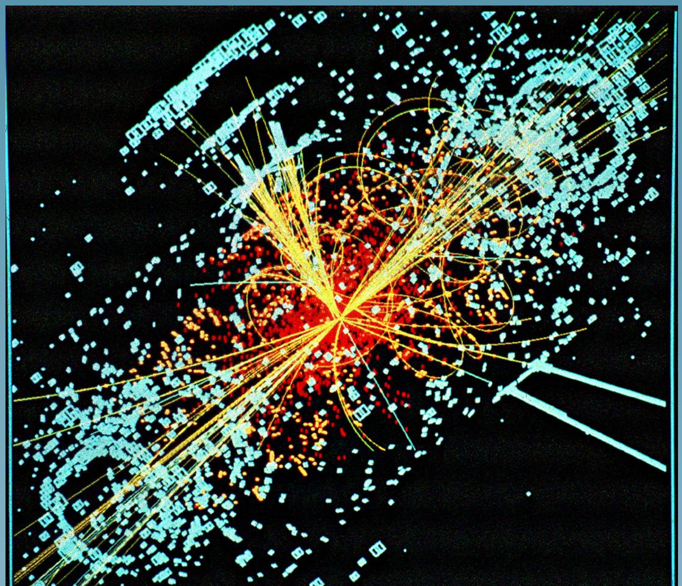

❀ QCD factorization theorem: σ for large momentum transfer is the convolution of the parton distribution functions with the parton level cross section. See Swain's lectures.

$$\sigma(AB \to F\ X) = \sum_{a,b} \int dx_1 dx_2\ f_{a/A}(x_1, Q^2) f_{b/B}(x_2, Q^2)\ \hat{\sigma}(ab \to F),$$

- $f_{b/B}(x, Q^2)dx$ is the number of partons carrying a fraction x of the hadron B momentum. Q^2 is a characteristic scale.

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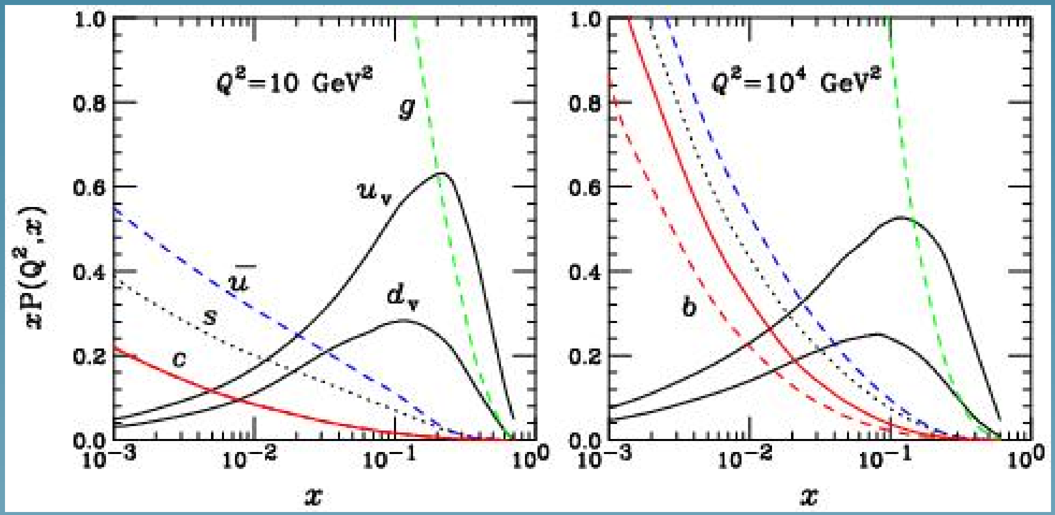
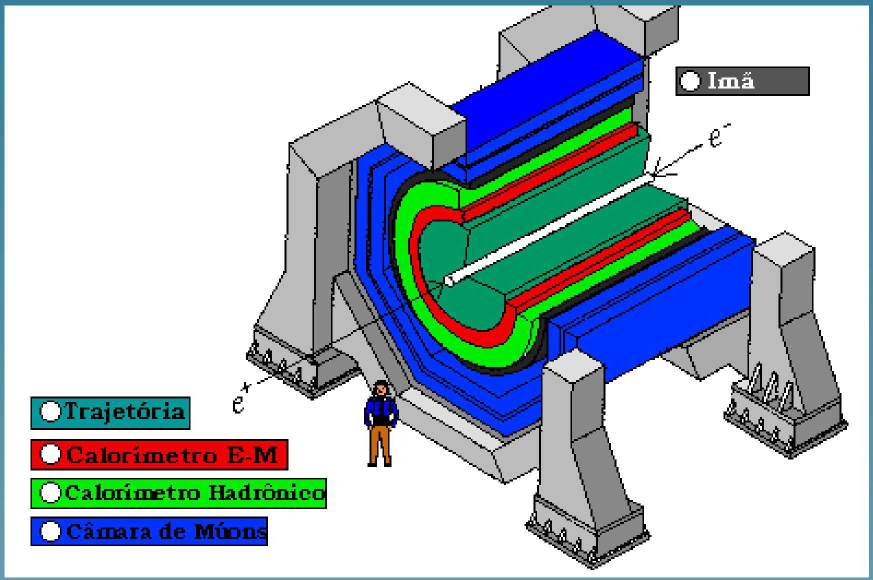


V. Detectors

✯ To gather information $\Longrightarrow$ modern detectors are very complex to measure position, time, momentum, energy, ...



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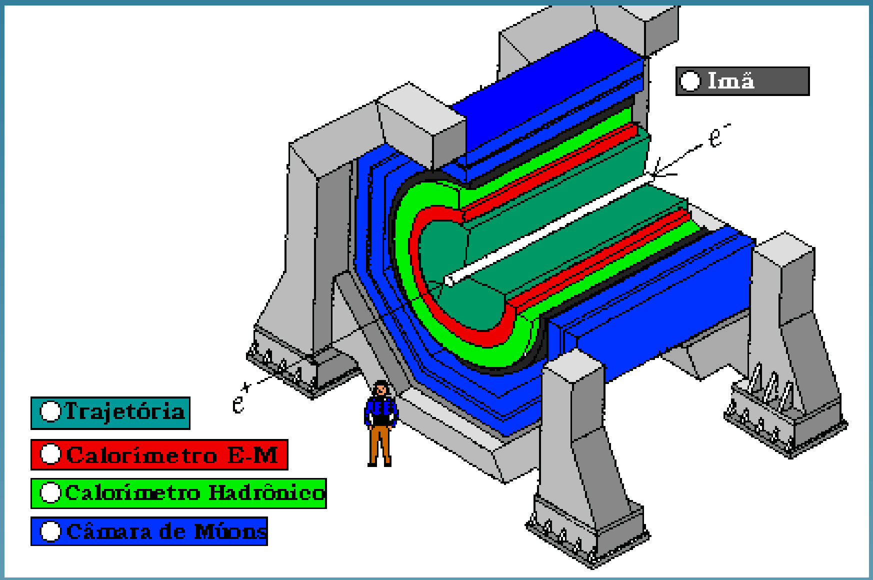
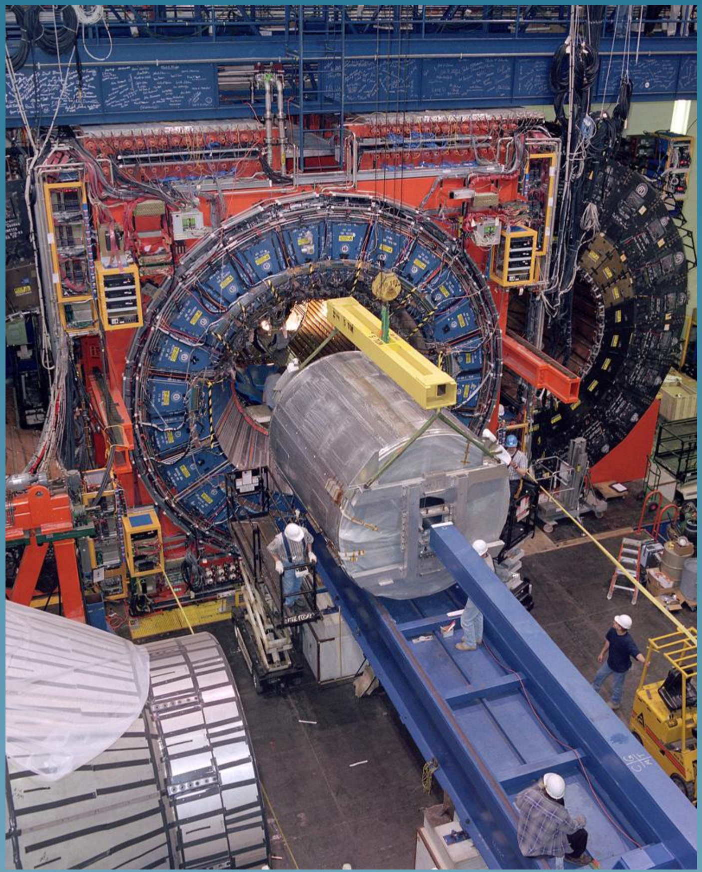




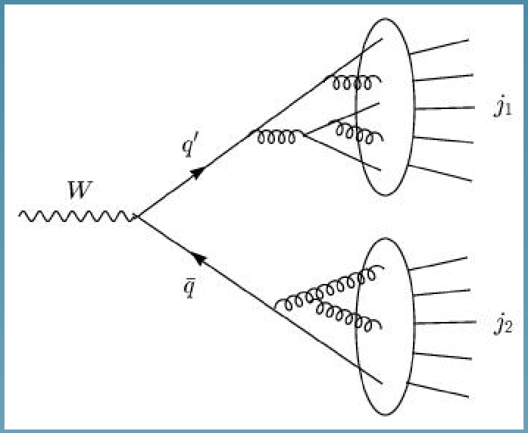
✯ The signal of a particle depends on its decay length

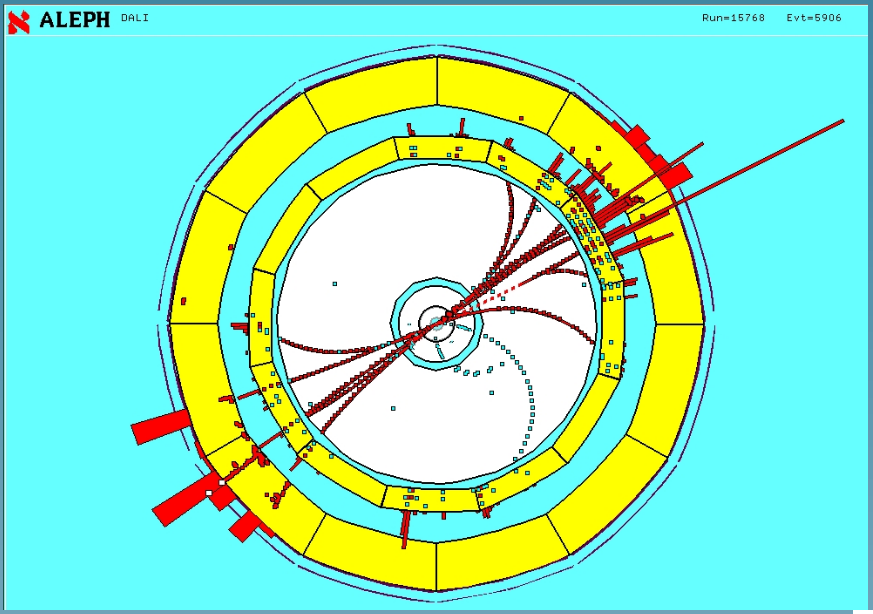
$$d = (\beta\ c\tau)\frac{E}{m} \approx (300\ \mu m)\left(\frac{\tau}{10^{-12}\ s}\right)\ \frac{E}{m},$$

✯ A few possibilities

- Quarks/gluons hadronize in $t_h \sim 1/\Lambda_{QCD} \approx 1/(200\ \ \text{MeV}) \approx 3.3 \times 10^{-24}$ s.

  energetic $\mathbf{q},\ \mathbf{g}$ give rise to jets





- Stable particles ($p, \; \bar{p}, \; e^{\pm}, \; \gamma$) will leave a footprint in the detector: energy deposity in CAL and/or hits on the tracking system

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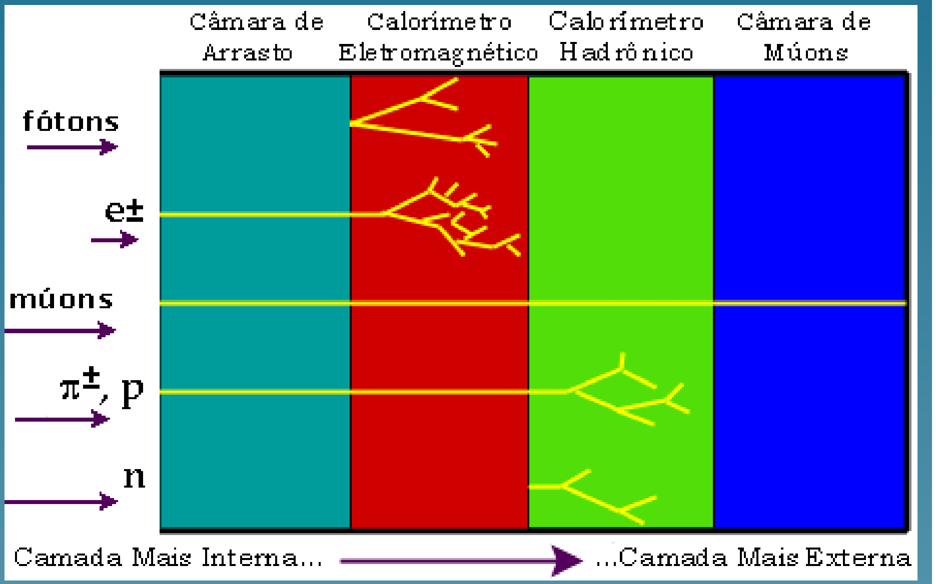
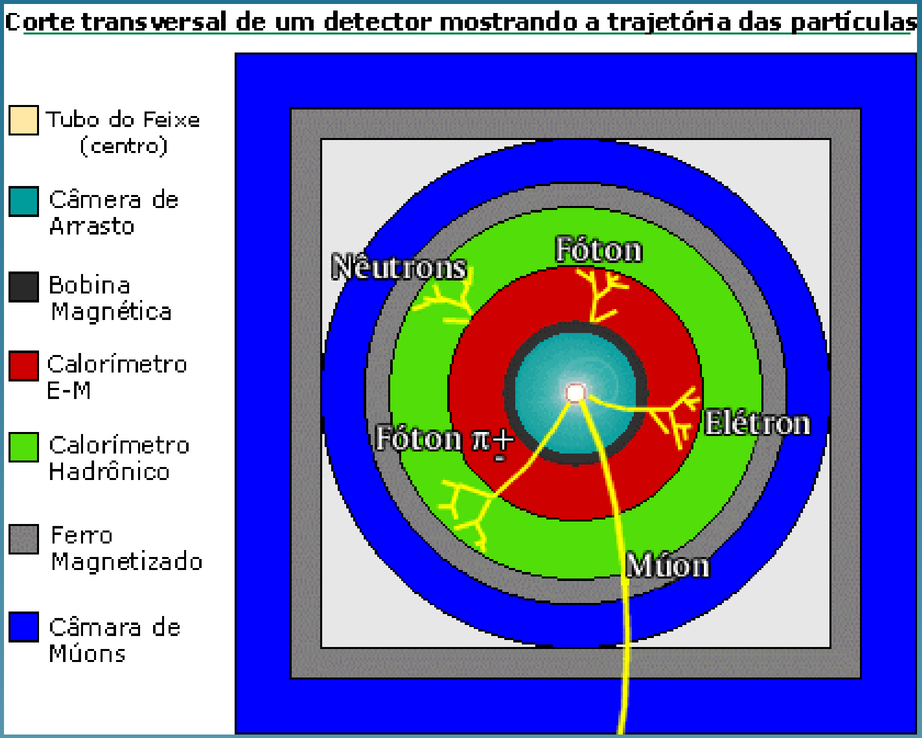
- displaced vertex: $B^{0,\pm},\ D^{0,\pm},\ \tau^{\pm}$, ($\tau \sim 10^{-12}$ s; $c\tau \sim 100\ \mu$m). $K^0_S \to \pi^+\pi^-$ with $c\tau \sim 2.7$ cm.

- Neutral particles with just weak interaction leave no signal in the detector (ν, LSP)

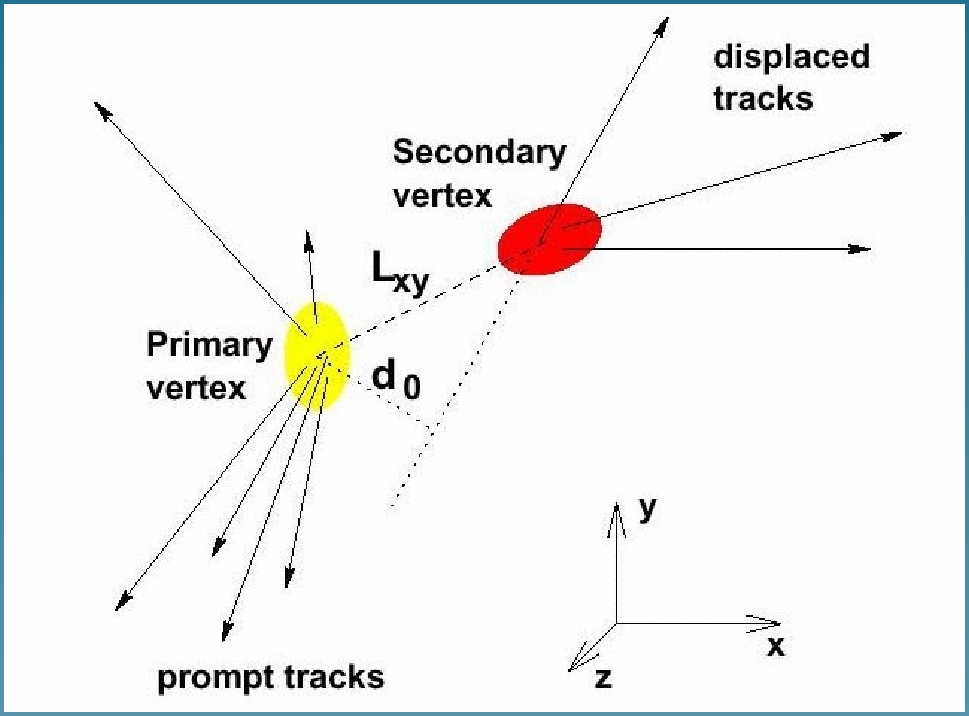
Leptons	Vertexing	Tracking	ECAL	HCAL	Muon Cham.
$e^{\pm}$	$\times$	$\vec{p}$	E	$\times$	$\times$
$\mu^{\pm}$	$\times$	$\vec{p}$	$\surd$	$\surd$	$\vec{p}$
$\tau^{\pm}$	$\surd\times$	$\surd$	$e^{\pm}$	$h^{\pm};\ 3h^{\pm}$	$\mu^{\pm}$
ν_e, ν_μ, ν_τ	$\times$	$\times$	$\times$	$\times$	$\times$
Quarks					
u, d, s	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$c \to D$	$\surd$	$\surd$	$e^{\pm}$	h's	$\mu^{\pm}$
$b \to B$	$\surd$	$\surd$	$e^{\pm}$	h's	$\mu^{\pm}$
$t \to bW^{\pm}$	b	$\surd$	$e^{\pm}$	$b + 2$ jets	$\mu^{\pm}$
Gauge bosons					
γ	$\times$	$\times$	E	$\times$	$\times$
g	$\times$	$\surd$	$\surd$	$\surd$	$\times$
$W^{\pm} \to \ell^{\pm}\nu$	$\times$	$\vec{p}$	$e^{\pm}$	$\times$	$\mu^{\pm}$
$\to q\bar{q}'$	$\times$	$\surd$	$\surd$	2 jets	$\times$
$Z^0 \to \ell^+\ell^-$	$\times$	$\vec{p}$	$e^{\pm}$	$\times$	$\mu^{\pm}$
$\to q\bar{q}$	$(b\bar{b})$	$\surd$	$\surd$	2 jets	$\times$

✯ Pictorically





✯ Typical uncertainties in measurements at LHC (ATLAS)



- Vertexing: impact parameter is an important handle
- $\Delta d_0 = 11 \oplus \frac{73}{(p_T/\text{ GeV})\ \sqrt{\sin\theta}}\ (\mu m)$,
- $\Delta z_0 = 87 \oplus \frac{115}{(p_T/\text{ GeV})\ \sqrt{\sin^3\theta}}\ (\mu m)$
- Tracking: $|\eta_\ell| \lesssim 2.5$ and $|\eta_h| \lesssim 5$
- ECAL: $\frac{\Delta E}{E} = \frac{10\%}{\sqrt{E/\text{GeV}}} \oplus 0.4\%$
- HCAL: $\frac{\Delta E}{E} = \frac{80\%}{\sqrt{E/\text{GeV}}} \oplus 15\%$

VI. Kinematics (at hadron colliders)

- Evaluating cross section in a hadron-hadron machine requires

$$\sigma = \int dx_1 dx_2 \sum_{\mathrm{subp}} f_{a_1/p}(x_1)\, f_{a_2/\bar{p}}(x_2)$$

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$$\frac{1}{2\hat{s}(2\pi)^{3n-4}} \int d\Phi_n(x_1 P_A + x_2 P_B;\; p_1 \dots p_n)\Theta(\text{cuts}) \overline{\sum} |\mathcal{M}|^2 (a_1 a_2 \to b_1 \dots b_n)$$

- the CM momentum $\simeq \sqrt{s}/2(x_1 + x_2, 0, 0, x_1 - x_2)$ (LAB $\neq$ CM)

- In general we can write $E(1, \beta \sin\theta \cos\phi, \beta \sin\theta \sin\phi, \beta \cos\theta)$, and

$$y \equiv \frac{1}{2} \log \frac{E + p_z}{E - p_z} \longrightarrow \eta = \frac{1}{2} \log \frac{1 + \cos\theta}{1 - \cos\theta} \quad \text{for} \quad \beta \to 1$$

rapidity differences are invariant under boosts along collision axis

- The CM and lab frames are related by

$$y = y^* + y_{c.m.} = y^* + \frac{1}{2} \log \frac{x_1}{x_2} \ ,$$

where $y_{c.m.}$ is the CM rapidity in the lab frame.

- Change of variables: $x_{1,2} = \sqrt{\tau}\ e^{\pm y_{cm}}$ that leads to

$$\int_{\tau_0}^{1} dx_1 \int_{\tau_0/x_1}^{1} dx_2 = \int_{\tau_0}^{1} d\tau \int_{\frac{1}{2}\ln\tau}^{-\frac{1}{2}\ln\tau} dy_{cm}$$

- The CM energy of the subprocesses is $\hat{s} = x_1 x_2 s = \tau s$

Phase space

❋ The sum of final states is

$$d\Phi_n(ab \to 1 \ldots n) \equiv \delta^4(p_a + p_b - p_1 - \ldots - p_n) \prod_{i=1}^{n} \frac{d^3\vec{p}_i}{2E_i}$$

❋ $3n - 4$ integrals.

❋ With azimuthal symmetry $\Longrightarrow$ $3n - 5$ integrals

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- In a hadron collider we have 2 extra integrals ($x_{1,2}$)

n	$3n-3$
2	3
3	6
4	9
...	...
8	21

Two-body kinematics

- The two-body phase space is (Show it!)

$$
\begin{aligned}
d\Phi_2 &\equiv \delta^4\left(P - p_1 - p_2\right) \frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2} \\
&= \frac{1}{4}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\, d\Omega_1 = \frac{1}{4}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\, d\cos\theta_1 d\phi_1 .
\end{aligned}
$$

where

$$
|\vec{p}_1^{\,cm}| = |\vec{p}_2^{\,cm}| = \frac{\lambda^{1/2}(s, m_1^2, m_2^2)}{2\sqrt{s}},\; E_1^{cm} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}},\; E_2^{cm} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}},
$$

with $\lambda(x, y, z) = (x - y - z)^2 - 4yz = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$

- The Mandelstan variables for $2 \to 2$ scattering process $p_a + p_b \to p_1 + p_2$ are

$$
\begin{aligned}
\hat{s} &= (p_a + p_b)^2 = (p_1 + p_2)^2 = E_{cm}^2, \\
\hat{t} &= (p_a - p_1)^2 = (p_b - p_2)^2 = m_a^2 + m_1^2 - 2(E_a E_1 - p_a p_1 \cos\theta_{a1}), \\
\hat{u} &= (p_a - p_2)^2 = (p_b - p_1)^2 = m_a^2 + m_2^2 - 2(E_a E_2 - p_a p_2 \cos\theta_{a2}).
\end{aligned}
$$

with $\hat{s} + \hat{t} + \hat{u} = ma^2 + m_b^2 + m_1^2 + m_2^2$.

- We can also write

$$
d\Phi_2 = \frac{1}{4} \, \frac{dt \; d\phi_1}{s \; \lambda^{1/2} \, (1, m_a^2/s, m_b^2/s)}.
$$

Three-body kinematics

- The three-body phase space is

$$d\Phi_3(ab \to 123) \equiv \delta^4 \left(p_a + p_b - p_1 - p_2 - p_3\right) \frac{d^3\vec{p}_1}{2E_1} \frac{d^3\vec{p}_2}{2E_2} \frac{d^3\vec{p}_3}{2E_3}$$

- One choice is to write as a chain of two-body PS (Show it!)

$$d\Phi_3(ab \to 123) = d\Phi_2(ab \to 1X) \ \times \ dM_X^2 \ \times \ d\Phi_2(X \to 23)$$

where X is the composite system $2+3$ and $M_X^2 = (p_2 + p_3)^2$

- Another possibility is

$$d\Phi_3(ab \to 123) = \frac{d^3\vec{p}_1}{2E_1} \ \times \ d\Phi_2(a + b - 1 \to 2 + 3)$$

- We can write

$$\frac{d^3\vec{p}}{E} = dp_x dp_y \frac{dp_z}{E} = p_T dp_T d\phi \, \frac{dp_z}{E}$$

or writing the momentum as a function of the rapidity

$$p^\mu = (E_T \cosh\eta, p_T \sin\phi, p_T \cos\phi, E_T \sinh\eta), \quad \text{with} \quad E_T = \sqrt{p_T^2 + m^2}$$

the phase space element then can be expressed as

$$\frac{d^3\vec{p}}{E} = p_T dp_T d\phi \; d\eta = E_T dE_T d\phi \; d\eta$$

Multiparticle Phase Space

- The choice of variables is dictated by the process being analyzed,

- For instance, $u\bar{u} \to g(p_1) + Z(\to e^+(p_2) + e^-(p_3)$. There is a peak on $m_{23}^2 = (p_2 + p_3)^2$, so m_{23} should be one of the integration variables.

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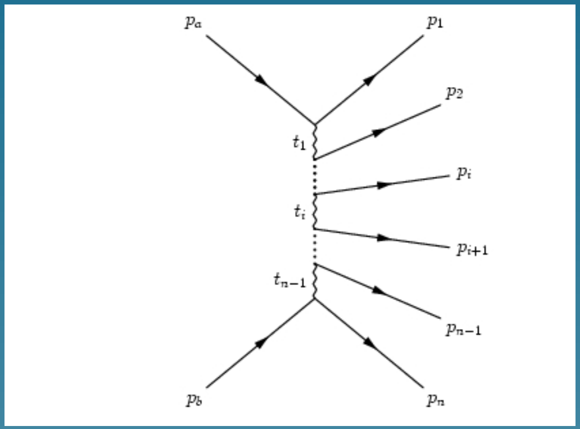
- General recursion relation: let $X = 1 + \ldots + j$ and $Y = (j+1) + \ldots + n$ then

$$d\Phi_n(ab \to 1 \ldots n) = d\Phi_2(ab \to XY) \times dM_X^2 \, dM_Y^2 \times d\Phi_j(X \to 1 \ldots j) \times d\Phi_{n-j}(Y \to j+1 \ldots n)$$

- It is possible to generate "generic" phase spaces

t-channel 

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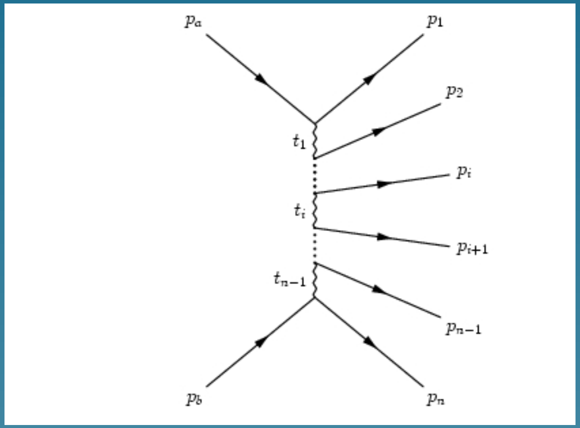
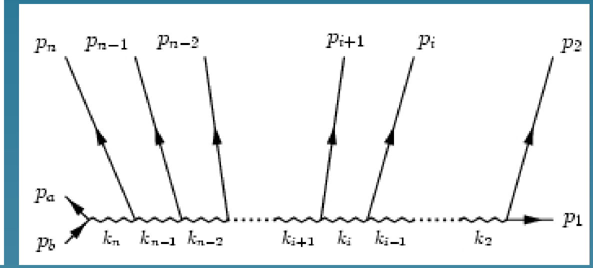
t-channel 



s-channel

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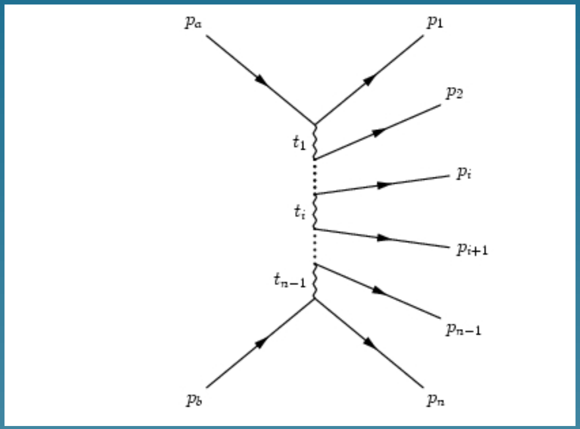
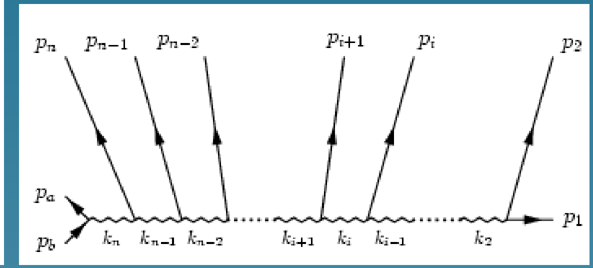
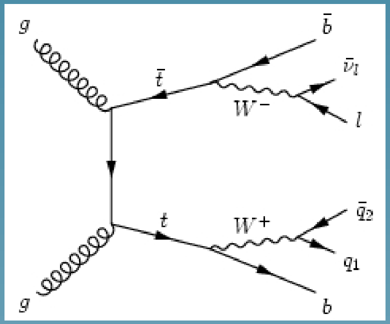




s-channel

- Another example



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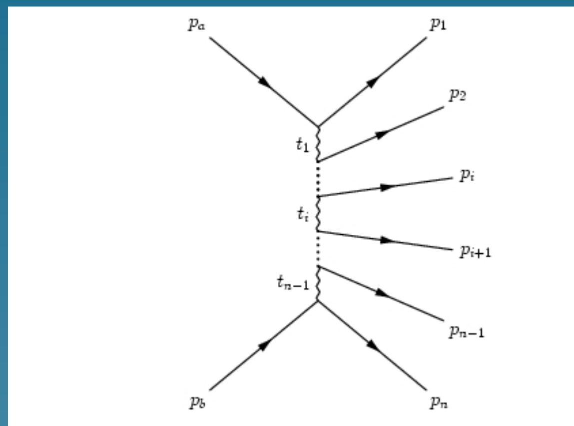
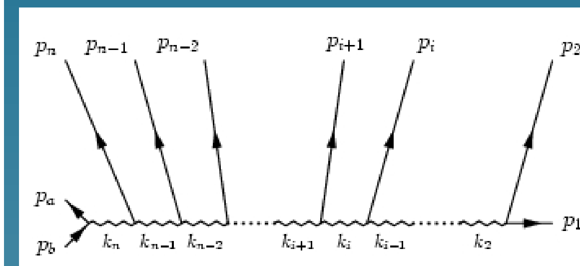
t-channel 



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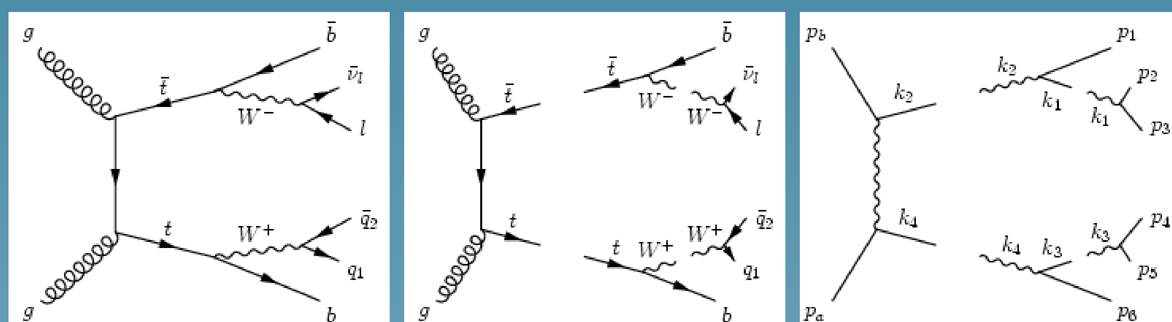

VII. Evaluation of scattering amplitudes

❄ We also need to evaluate $\overline{\sum}|\mathcal{M}|^2(a_1a_2 \to b_1 \dots b_n)$ with $\mathcal{M} = \sum_{i=1}^{f} \mathcal{M}_i$.

❄ If f (n) is large the "trace technique" becomes useless since we have to evaluate $f(f+1)/2$ cross terms Re($\mathcal{M}_i^* \mathcal{M}_j$).

❄ It then becomes advantageous to numerically evaluate $\mathcal{M}_i \implies$ complexity grows linearly with f.

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- For fermions

in the representation $\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ we write $\psi = \begin{pmatrix} \psi_- \\ \psi_+ \end{pmatrix}$

where ψ_- and ψ_+ are Weyl spinors of negative and positive helicity.

❄ For instance, u-spinor with chiral components $u(p,\sigma)_\pm = \sqrt{p^0 \pm \sigma|\mathbf{p}|}\;\chi_\sigma(p)$, where

$$\chi_+(p) = \frac{1}{\sqrt{2|\mathbf{p}|(|\mathbf{p}|+p_z)}} \begin{pmatrix} |\mathbf{p}|+p_z \\ p_x+ip_y \end{pmatrix} \;\; ; \;\; \chi_-(p) = \frac{1}{\sqrt{2|\mathbf{p}|(|\mathbf{p}|+p_z)}} \begin{pmatrix} -p_x+ip_y \\ |\mathbf{p}|+p_z \end{pmatrix}$$

❄ The HELAS package has all elements need to evaluate Feynman diagrams defined as fortran routines. For instance, an incoming $u(p,NH)$-spinor is given by a simple subroutine call,

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call IXXXXX(P,FMASS,NH,+1,PSI)
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❄ Outgoing spinors are generate by `call IXXXXX(P, FMASS, NH, ±1, PSI)`

❄ the polarization vector of incoming vector bosons is `call VXXXXXX(P, VMASS, NHEL, −1, VC)`

❄ The HELAS system contains external line routines

External line	Subroutine
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Out Fermion	OXXXXX
Vector Boson	VXXXXX
Scalar Boson	SXXXXX

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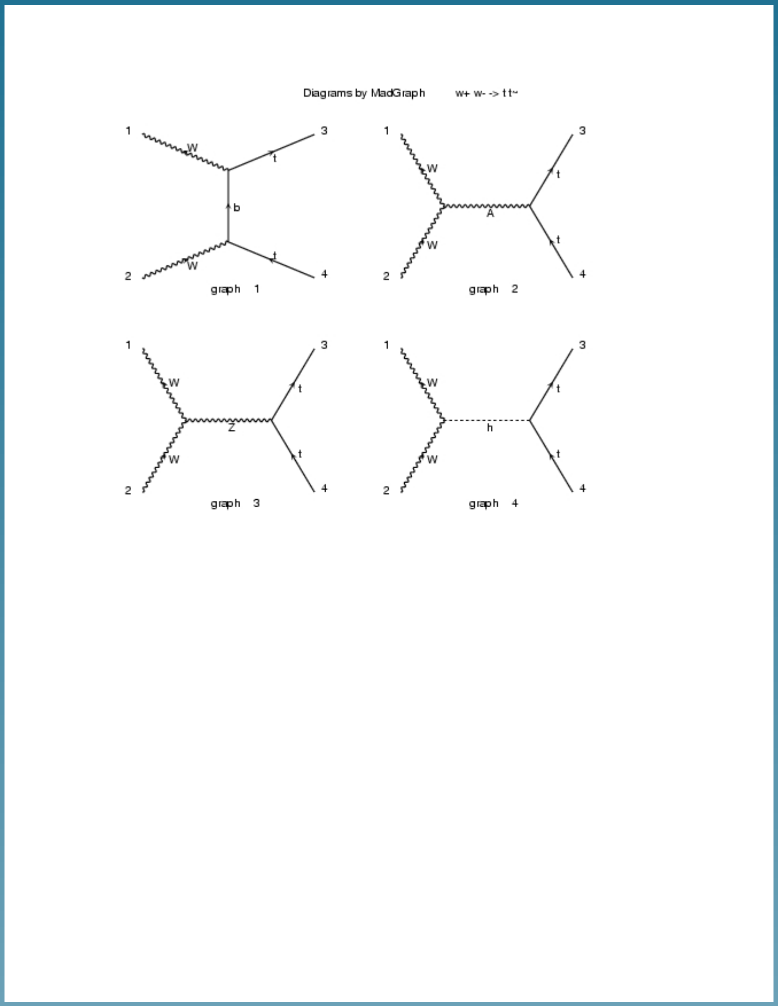
- as well as the renormalizable vertices

Vertex	Inputs	Output	Subroutine
FFV	FFV	Amplitude	IOVXXX
	FF	V	JIOXXX, J3XXXX
	FV	F	FVIXXX, FVOXXX
FFS	FFS	Amplitude	IOSXXX
	FF	S	HIOXXX
	FS	F	FSIXXX, FSOXXX
VVV	VVV	Amplitude	VVVXXX
	VV	V	JVVXXX
VVS	VVS	Amplitude	VVSXXX
	VS	V	JVSXXX
	VV	S	HVVXXX
VSS	VSS	Amplitude	VSSXXX
	SS	V	JSSXXX
	VS	S	HVSXXX
SSS	SSS	Amplitude	SSSXXX
	SS	S	HSSXXX
VVVV	VVVV	Amplitude	WWWWXX, W3W3XX
	VVV	V	JWWWXX, JW3WXX
VVSS	VVSS	Amplitude	VVSSXX
	VSS	V	JVSSXX
	VVS	S	HVVSXX
SSSS	SSSS	Amplitude	SSSSXX
	SSS	S	HSSSXX

❄ Example: $W^+W^- \to t\bar{t}$



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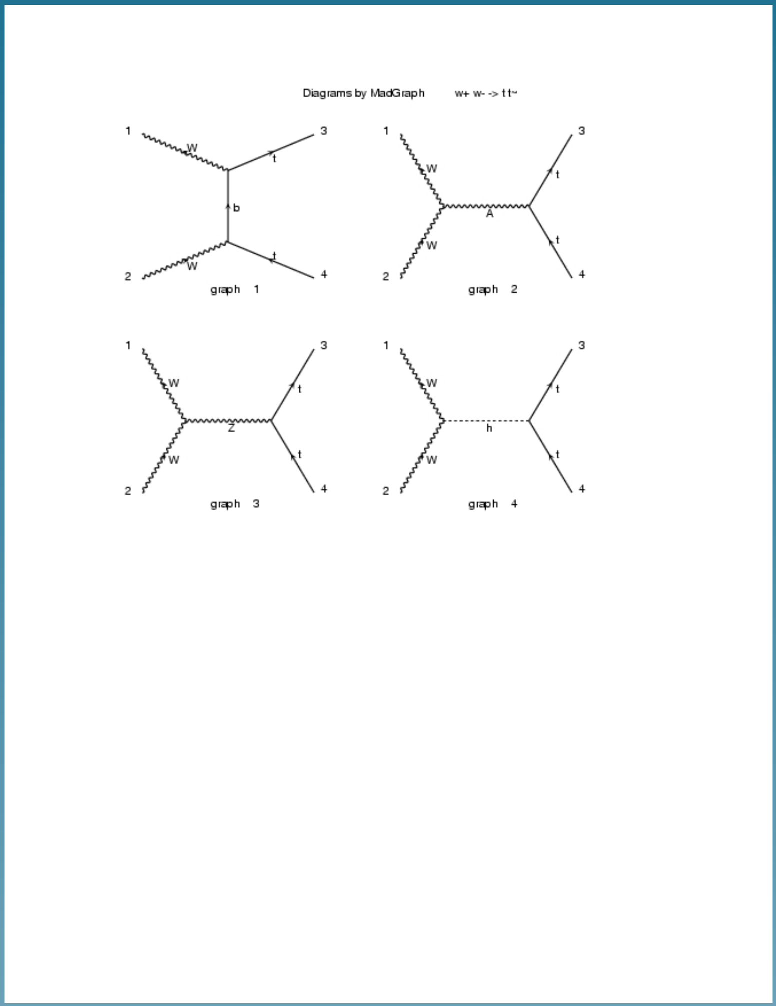


```
C...external lines
C
      CALL VXXXXX(PWM,WMASS,NHWM,-1 , WM)
      CALL VXXXXX(PWP,WMASS,NHWP,-1 , WP)
      CALL OXXXXX(PT ,TMASS,NHT ,+1 , FO)
      CALL IXXXXX(PTB,TMASS,NHTB,-1 , FI)
C
C...evaluating the Feynman diagrams
C
      CALL J3XXXX(FI,FO,GAU,GZU,ZMASS,ZWIDTH , J3)
      CALL VVVXXX(WP,WM,J3,GW , AMPS)
      CALL FVIXXX(FI,WM,GWF,0.,0. , FVI)
      CALL IOVXXX(FVI,FO,WP,GWF , AMPT)
      CALL HIOXXX(FI,FO,GCHT,HMASS,HWIDTH , HTT)
      CALL VVSXXX(WM,WP,HTT,GWWH , AMPH)
```

❄ The package MADGRAPH can be used to generate SM and SUSY amplitudes!

- MADGRAPH can generate $2 \to 8$ processes
- MADGRAPH already sums over polarizations and colors
- MADGRAPH produces a ps file with the Feynman diagrams
- The package MADEVENT goes further and produces a complete Monte Carlo
- Interfaces for PYTHIA, HERWIG, and ROOT are available

`http://madgraph.hep.uiuc.edu/`

❄ There are many more packages that evaluate scattering amplitudes:

- CompHEP is a complete system for unpolarized scattering of up to four particles in the final state. It features convenient diagram generation and calculation of the squared matrix element as well as efficient Monte Carlo integration.

- ALPHA implements an efficient algorithm for calculating Born amplitudes for many particles in the final state.

- O'Mega is an optimizing matrix element compiler.

- WHIZARD calls O'Mega, CompHEP, or MADGRAPH to calculate complete matrix elements matrix elements and creates an unweighted event generator.

Monte Carlo integration

- To obtain σ we need to evaluate $\mathbf{N_{int} = 3n - 2}$ integrals.

- The uncertainty of numerical methods grows fast with $\mathbf{N_{int}}$ for K evaluations of the integrand

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Method	1-d uncertainty	$\mathbf{N_{int}}$ uncertainty
trapezoidal rule	K^{-2}	$K^{-2/\mathbf{N_{int}}}$
Simpson	K^{-4}	$K^{-4/\mathbf{N_{int}}}$
Gauss	K^{-2m+1}	$K^{-(2m+1)/\mathbf{N_{int}}}$

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Monte Carlo	$K^{-1/2}$	$K^{-1/2}$

- It is usually advantageous to use Monte Carlo.

Basic idea:

- Initially we map the integration region into a $\mathbf{3n-2}$ dimensional hypercube $(\mathbf{0 \leq r_i \leq 1})$

$$\mathbf{dx_1 dx_2 d\Phi_n = J \prod_{i=1}^{3n-2} dr_i}$$

- Now take K sets $\{\mathbf{r_i}\}$ of $\mathbf{3n-2}$ random numbers each and a good approximation is

$$\sigma \approx \frac{\mathbf{1}}{\mathbf{K}} \sum_{\{\mathbf{r_i}\}} \frac{\mathbf{J}}{\mathbf{2\hat{s}(2}\pi\mathbf{)^{3n-4}}} \sum_{\mathbf{subprocesses}} \mathbf{f(x_1)f(x_2)}\overline{\sum} |\mathcal{M}|^{\mathbf{2}}\, \mathbf{\Theta}(\mathrm{cuts}) \;,$$

which gives the correct limit for $K \to \infty$

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❋ The variance of $I = \int dx g(x)$ is $\int dx (I - g(x))^2 / \sqrt{K}$ (VEGAS, BASES, etc)

- Example: $\mathbf{e^+e^-} \rightarrow \mathbf{2}$ particles in the final state

$$d\Phi_2 = \frac{1}{4}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}}\, d\cos\theta_1 d\phi_1 = \frac{1}{4}\frac{|\vec{p}_1^{\,cm}|}{\sqrt{s}} \times 4\pi \times dr_1 dr_2$$

with $\cos\theta_1 = -1 + 2r_1$ and $\phi_1 = 2\pi r_2$. More, I can construct the (massless) momentum with this

$$p_1 = \frac{\sqrt{s}}{2}\,(1,\ \sin\theta_1\cos\phi_1,\ \sin\theta_1\sin\phi_1,\ \cos\theta_1)$$

$$p_2 = \frac{\sqrt{s}}{2}\,(1,\ -\sin\theta_1\cos\phi_1,\ -\sin\theta_1\sin\phi_1,\ -\cos\theta_1)$$

✺ General comments:

- The change of variable should produce a constant function to reduce the fluctuations.
- It is easy to construct the momenta p_i and check the acceptance cuts $\Theta(cuts)$;
- Each event (set of random numbers) is associated to a weight factor

$$\mathbf{w} = \sum_{\{\mathbf{r_i}\}} \frac{\mathbf{J}}{\mathbf{2\hat{s}(2\pi)^{3n-4}}} \sum_{\mathbf{subprocesses}} \mathbf{f(x_1)f(x_2)}\overline{\sum}|\mathcal{M}|^{\mathbf{2}}\, \boldsymbol{\Theta}(\mathrm{cuts}) \ ,$$

- It is easy to generate distributions $d\sigma/dz$: Given the bin width Δz one adds $w \cdot wgt/\Delta z$ to the bin corresponding to z. $wgt \simeq 1/K$
- It is possible to generate unweighted events.